

# Degrees of Freedom of Multi-hop MIMO Broadcast Networks with Delayed CSIT

Zhao Wang, Ming Xiao, Chao Wang, and Mikael Skoglund

## Abstract

We study the sum degrees of freedom (DoF) of a class of multi-layer relay-aided MIMO broadcast networks with *delayed* channel state information at transmitters (CSIT). In the assumed network a  $K$ -antenna source intends to communicate to  $K$  single-antenna destinations, with the help of  $N - 2$  layers of  $K$  full-duplex single-antenna relays. We consider two practical delayed CSIT feedback scenarios. If the source can obtain the CSI feedback signals from all layers, we prove the optimal sum DoF of the network to be  $\frac{K}{1 + \frac{1}{2} + \dots + \frac{1}{K}}$ . If the CSI feedback is only within each hop, we show that when  $K = 2$  the optimal sum DoF is  $\frac{4}{3}$ , and when  $K \geq 3$  the sum DoF  $\frac{3}{2}$  is achievable. Our results reveal that the sum DoF performance in the considered class of  $N$ -layer MIMO broadcast networks with delayed CSIT may depend not on  $N$ , the number of layers in the network, but only on  $K$ , the number of antennas/terminals in each layer.

## Index Terms

Degrees of freedom, multi-hop MIMO broadcast network, delayed CSIT, interference alignment.

## I. INTRODUCTION

With the increasing interest in deploying relays in 4th generation mobile networks, multi-user multi-hop systems have drawn substantial research attention. In spite of the rapid advances in the understanding of single-hop networks, our knowledge on how to deal with inter-user interference and design efficient transmission schemes in multi-hop systems is relatively limited. For instance, we consider a wireless communication system in which a  $K$ -antenna source intends

The authors are with the Communication Theory Lab., School of Electrical Engineering, Royal Institute of Technology (KTH), Stockholm, Sweden (E-mail: {zhaowang, mingx, chaowang, skoglund}@kth.se).

to communicate to  $K$  single-antenna destinations. If the source's transmission can directly reach the destinations, this system is a well-studied  $K$ -user MIMO broadcast channel. It is already known that if perfect channel state information at transmitter (CSIT) is available, the optimal sum degrees of freedom (DoF) of the system is  $K$ , while without CSIT the result is only one. Clearly, CSIT serves as a very important factor that influences system capacity. In practice, channel estimation is in general performed by receivers and CSIT is typically obtained via feedback signals sent from them. However, attaining perfect *instantaneous* CSIT in realistic systems may be a challenging task when feedback delay is not negligible compared with channel coherence time. To gain understanding in such scenarios, Maddah-Ali and Tse [1] proposed a *delayed CSIT* concept to model the extreme case where channel coherence time is smaller than feedback delay so that CSIT would be completely outdated. They showed that by interference alignment (IA) design even the outdated CSIT can be advantageous to offer DoF gain achieving the optimal sum DoF of a  $K$ -user MIMO broadcast channel  $\frac{K}{1+\frac{1}{2}+\dots+\frac{1}{K}}$  [1]. Hence, from a DoF perspective, communication in this single-hop network is relatively well understood.

Nevertheless, if the source and the destinations are not physically connected so that the communication has to be assisted by intermediate relays, how many DoF are available is not clear, especially when potentially *multiple layers* of relays are required and only delayed CSIT can be available. To study the DoF of a multi-hop network, a straightforward *cascade approach* sees the network as a concatenation of individual single-hop sub-networks. The network DoF is limited by the minimum DoF of all sub-networks. In this paper, we consider a class of relay-aided MIMO broadcast networks with a  $K$ -antenna source,  $K$  single-antenna destinations, and  $N - 2$  relay layers, each containing  $K$  single-antenna full-duplex relays. Following the cascade approach, the first hop can be treated as a  $K$ -user MIMO broadcast channel. Each of the remaining hops can be seen as a  $K \times K$  single-antenna X channel [2]. Hence, the achievable sum DoF of the considered network is  $\frac{4}{3} - \frac{2}{3(3K-1)}$ , i.e. that of a  $K \times K$  X channel [3].

However, separating the network into individual sub-networks may not always be a good strategy. For instance, provided perfect instantaneous CSIT, references [4]–[6] showed that in certain systems designing transmission by treating all hops as a whole entity can perform strictly better than applying the cascade approach. In this paper, we will show that with delayed CSIT this is also the case for the considered  $N$ -layer relay-aided MIMO broadcast networks. Specifically, we focus on two delayed CSIT scenarios. In a *global-range* feedback scenario, where the CSI

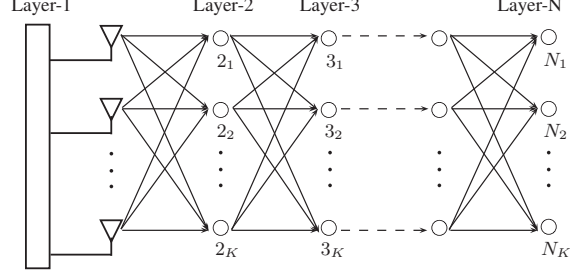


Fig. 1.  $(N, K)$  relay-aided MIMO broadcast networks.

of all layers can be decoded by the source, we propose a joint transmission design to prove the optimal network sum DoF to be  $\frac{K}{1+\frac{1}{2}+\dots+\frac{1}{K}}$ . In addition, in a *one-hop-range* feedback scenario, where the CSI feedback signals sent from each layer can only be received by its adjacent upper-layer, we show that when  $K = 2$  the optimal sum DoF  $\frac{4}{3}$  is achievable, and when  $K \geq 3$  a DoF  $\frac{3}{2}$  is achievable. These results depend not on  $N$  but only on  $K$ , and are clearly better than those attained by the cascade approach.

## II. SYSTEM MODEL

As shown in Fig. 1, we consider a multi-hop MIMO broadcast network in which a source node with  $K$  transmit antennas intends to communicate to  $K$  single-antenna destinations. There is no physical link between them so that  $N - 2$  ( $N \geq 3$ ) layers of intermediate relay nodes, each with  $K$  full-duplex single-antenna relays, are deployed to aid the communication. The network contains a total of  $N$  layers of nodes. No connection exists between non-adjacent layers. We term this network an  $(N, K)$  relay-aided MIMO broadcast network throughout the paper.  $n_k$  is used to represent the node  $k$  ( $k \in \{1, 2, \dots, K\}$ ) at layer- $n$  ( $n \in \{2, 3, \dots, N\}$ ).

Assume the rate tuple  $(R_1, R_2, \dots, R_K)$  between the source and destinations can be achieved. Let  $\mathcal{C}$  denote the capacity region and  $P$  denote the power constraint of each layer. The sum DoF of the  $(N, K)$  relay-aided MIMO broadcast network with delayed CSIT is defined as [1]

$$D^{d-CSI}(N, K) = \max_{(R_1, \dots, R_K) \in \mathcal{C}} \left\{ \lim_{P \rightarrow \infty} \frac{\sum_{i=1}^K R_i(P)}{\log P} \right\}. \quad (1)$$

Let a  $K \times K$  matrix  $\mathbf{H}^{[n-1]}(t)$  denote the channel matrix between the  $(n-1)$ th and the  $n$ th layers (i.e. the  $(n-1)$ th hop) at time slot  $t$ . The  $i$ th row and  $k$ th column element of  $\mathbf{H}^{[n-1]}(t)$ ,  $h_{ik}^{[n-1]}(t)$ , represents the channel gain from node  $(n-1)_k$  to node  $n_i$ . We consider block fading

channels. All fading coefficients remain constant within one time slot, but change independently across different time slots. Let  $x_k^{[n-1]}(t)$  ( $E[|x_k^{[n-1]}(t)|^2] \leq \frac{P}{K}$ ) and  $y_k^{[n]}(t)$  represent the transmit signal of node  $(n-1)_k$  and the received signal of node  $n_k$  at time slot  $t$ , respectively. The received signals of layer- $n$  is

$$\mathbf{y}^{[n]}(t) = \mathbf{H}^{[n-1]}(t)\mathbf{x}^{[n-1]}(t) + \mathbf{z}^{[n]}(t), n = 2, 3, \dots, N, \quad (2)$$

where  $\mathbf{x}^{[n-1]}(t) = [x_1^{[n-1]}(t) \ x_2^{[n-1]}(t) \ \dots \ x_K^{[n-1]}(t)]^T$  is the transmit signals of layer- $(n-1)$ ,  $\mathbf{y}^{[n]}(t) = [y_1^{[n]}(t) \ y_2^{[n]}(t) \ \dots \ y_K^{[n]}(t)]^T$ , and  $\mathbf{z}^{[n]}(t)$  is the unit-power complex additive white Gaussian noise (AWGN).

At each time slot  $t$ , each receiver is able to obtain the CSI of its incoming channels by a proper training process. That is,  $n_i$  knows  $h_{ik}^{[n-1]}(t)$ ,  $\forall k \in \{1, 2, \dots, K\}$ . Such knowledge can be directly delivered to nodes in later layers along with data transmission. To transmit CSI to previous layers, feedback signals are used from each receiver. We assume that the feedback delay is larger than the channel coherence time. Thus if any transmitter can receive and decode the feedback signals, its obtained CSIT is in fact delayed by one time slot. In this paper, we consider two scenarios of delayed CSIT feedback in the  $(N, K)$  relay-aided MIMO broadcast network:

1) *Global-range delayed CSIT*: In this scenario, the source node can receive and successfully decode the feedback signals transmitted by all nodes. Hence it can obtain the global CSI  $\mathbf{H}^{[1]}(t), \mathbf{H}^{[2]}(t), \dots, \mathbf{H}^{[N-1]}(t)$  at time slot  $t + 1$ .

2) *One-hop-range delayed CSIT*: In this case, the feedback signals can be delivered only between adjacent layers. Then at time slot  $t + 1$ ,  $\mathbf{H}^{[n-1]}(t)$  is known at only layer- $(n-1)$ .

### III. MAIN RESULTS AND DISCUSSIONS

We study the sum DoF of the considered  $(N, K)$  relay-aided MIMO broadcast network, for both global-range and one-hop-range delayed CSIT scenarios. Our main results are summarized in the following two theorems.

*Theorem 1*: With *global-range* delayed CSIT, the sum DoF of the  $(N, K)$  relay-aided MIMO broadcast network is

$$D^{d-CSI}(N, K) = \frac{K}{1 + \frac{1}{2} + \dots + \frac{1}{K}}. \quad (3)$$

*Proof*: Please see Section IV for the proof. ■

*Theorem 2:* With *one-hop-range* delayed CSIT, the sum DoF of the  $(N, K)$  relay-aided MIMO broadcast network is

$$D^{d-CSI}(N, 2) = \frac{4}{3},$$

$$\frac{3}{2} \leq D^{d-CSI}(N, K) \leq \frac{K}{1 + \frac{1}{2} + \dots + \frac{1}{K}}, \quad K \geq 3. \quad (4)$$

*Proof:* Please see Section V for the proof. ■

We can see that  $N$  does not appear in (3) or (4). Thus, the sum DoF of the  $(N, K)$  relay-aided broadcast network would not be limited by the number of layers in the network, but may be related only to  $K$ , the number of antennas/users.

With *global-range* feedback, the sum DoF of the network is the same as that in a single-hop  $K$ -user MIMO broadcast channel. The result reveals the importance of providing the CSI of the whole network to the source. In practice, this can be achieved by e.g., each node broadcasting its feedback signal with a sufficiently high power. However, this may not be possible in some systems, and *one-hop-range* feedback may be more feasible. In this case, the CSI flow is limited within only one hop, which in turn affects the interference management in the network. When  $K=2$ , the sum DoF is shown to be  $\frac{4}{3}$ , by a joint transmission design among all hops. Following a similar strategy, the sum DoF can be lower bounded by  $\frac{3}{2}$  for  $K \geq 3$ . Although currently it is difficult to quantify the distance between this lower bound and the actual achievable sum DoF, we can see that when  $K$  is small, e.g.,  $K = 3, 4$ , the lower bound is tight since it is only slightly smaller than a sum DoF upper bound.

Recall that applying the cascade approach the achievable sum DoF is limited by that of a  $K \times K$  X channel, i.e.,  $\frac{4}{3} - \frac{2}{3(3K-1)}$  [3]. By a joint transmission design among all hops, our scheme strictly surpasses the cascade approach. The task of proving the optimality of our results or finding even better schemes to attain the actual sum DoF of an  $(N, K)$  relay-aided MIMO broadcast network will be left for future investigation.

#### IV. PROOF OF THEOREM 1

*1) Outer Bound:* We assume that all the relays in each layer can fully cooperate and jointly process their signals. Since this assumption would not reduce network performance, the sum DoF of this new system, which is clearly limited by that of the last hop (i.e. a single-hop  $K$ -user

MIMO broadcast channel), would serve as an outer bound of the sum DoF of the considered  $(N, K)$  relay-aided broadcast network. According to [1], the outer bound is  $\frac{K}{1+\frac{1}{2}+\dots+\frac{1}{K}}$ .

2) *Achievability*: Consider full-duplex amplify-and-forward relays. At time slot  $t$ , node  $n_i$  chooses  $g_i^{[n]}(t)$  as its amplification coefficient such that  $|g_i^{[n]}(t)|^2 \left( \sum_{k=1}^K |h_{ik}^{[n-1]}(t)|^2 + \frac{1}{K} \right) \leq 1$ . We define  $\mathbf{G}^{[n]}(t) = \text{diag}\{g_1^{[n]}(t), g_2^{[n]}(t), \dots, g_K^{[n]}(t)\}$  and focus on the high-SNR regime (where DoF is effective). Hence we omit the noise term in (2). At time slot  $t$ , the received signals at the layer- $N$  (i.e. the destinations) can be denoted by

$$\mathbf{y}^{[N]}(t) = \left( \prod_{n=3}^N \mathbf{H}^{[n-1]}(t) \mathbf{G}^{[n-1]}(t) \right) \mathbf{H}^{[1]}(t) \mathbf{x}^{[1]}(t). \quad (5)$$

Let  $\tilde{\mathbf{H}}(t) = \prod_{n=3}^N (\mathbf{H}^{[n-1]}(t) \mathbf{G}^{[n-1]}(t)) \mathbf{H}^{[1]}(t)$  and substitute it into (5). We obtain an equivalent single-hop  $K$ -user MIMO broadcast channel with an equivalent channel matrix  $\tilde{\mathbf{H}}(t)$ . Because the source has the delayed CSI of the whole network,  $\tilde{\mathbf{H}}(t-1)$  is known at time slot  $t$ . Thus the transmission scheme proposed in [1] for achieving the sum DoF of a single-hop  $K$ -user MIMO broadcast channel with delayed CSIT can also be employed in the equivalent system. The sum DoF outer bound  $\frac{K}{1+\frac{1}{2}+\dots+\frac{1}{K}}$  is achievable.

## V. PROOF OF THEOREM 2

Clearly, the outer bound above still holds for *one-hop-range* feedback. When  $K = 2$  it can be shown that the outer bound  $\frac{4}{3}$  is tight. However, it may not be true for  $K \geq 3$ . In this proof we will present a new multi-round transmission scheme that treats all hops as a whole entity aiming for aligning interference. The achievable sum DoF is higher than that obtained by the cascade approach and thus will serve as a lower bound to the sum DoF of the considered network.

Due to the page limitation, we will mainly focus on an example  $(3, 3)$  relay-aided MIMO broadcast network. Let integer  $l \geq 1$ . We will show that  $9l$  independent messages can be delivered from the 3-antenna source to the 3 single-antenna destinations through a layer of 3 single-antenna full-duplex relays, using a total of  $6l + 3$  time slots. Then when  $l \rightarrow \infty$ , the sum DoF  $\frac{3}{2}$  can be asymptotically achieved. The corresponding approach for general networks will be given later.

Recall that we use  $y_k^{[n]}(t)$  and  $x_k^{[n]}(t)$  respectively to denote the received and transmitted signals of the  $k$ th node in layer- $n$  (or the  $k$ th antenna if  $n = 1$ ) at time slot  $t$ . The transmission process in the  $(3, 3)$  relay-aided MIMO broadcast network, for the first 12 time slots, is shown in Table I. Specifically, 2 rounds of messages, each containing 9 independent messages, are delivered to

TABLE I

$t$	1	2	3	4	5	6	7	8	9	10	11	12
$x_1^{[1]}(t)$	$\mu_1(1)$	$\mu_2(1)$	$\mu_3(1)$	$L_2^{[2]}(1)$	$L_3^{[2]}(1)$		$\mu_1(2)$	$\mu_2(2)$	$\mu_3(2)$	$L_2^{[2]}(2)$	$L_3^{[2]}(2)$	
$x_2^{[1]}(t)$	$\nu_1(1)$	$\nu_2(1)$	$\nu_3(1)$	$L_4^{[2]}(1)$		$L_6^{[2]}(1)$	$\nu_1(2)$	$\nu_2(2)$	$\nu_3(2)$	$L_4^{[2]}(2)$		$L_6^{[2]}(2)$
$x_3^{[1]}(t)$	$\omega_1(1)$	$\omega_2(1)$	$\omega_3(1)$		$L_7^{[2]}(1)$	$L_8^{[2]}(1)$	$\omega_1(2)$	$\omega_2(2)$	$\omega_3(2)$		$L_7^{[2]}(2)$	$L_8^{[2]}(2)$
$y_1^{[2]}(t)$	$L_1^{[2]}(1)$	$L_4^{[2]}(1)$	$L_7^{[2]}(1)$	$\gamma_{12}^{[2]}(1)$	$\gamma_{13}^{[2]}(1)$		$L_1^{[2]}(2)$	$L_4^{[2]}(2)$	$L_7^{[2]}(2)$	$\gamma_{12}^{[2]}(2)$	$\gamma_{13}^{[2]}(2)$	
$y_2^{[2]}(t)$	$L_2^{[2]}(1)$	$L_5^{[2]}(1)$	$L_8^{[2]}(1)$	$\gamma_{12}^{[2]}(1)$		$\gamma_{23}^{[2]}(1)$	$L_2^{[2]}(2)$	$L_5^{[2]}(2)$	$L_8^{[2]}(2)$	$\gamma_{12}^{[2]}(2)$		$\gamma_{23}^{[2]}(2)$
$y_3^{[2]}(t)$	$L_3^{[2]}(1)$	$L_6^{[2]}(1)$	$L_9^{[2]}(1)$		$\gamma_{13}^{[2]}(1)$	$\gamma_{23}^{[2]}(1)$	$L_3^{[2]}(2)$	$L_6^{[2]}(2)$	$L_9^{[2]}(2)$		$\gamma_{13}^{[2]}(2)$	$\gamma_{23}^{[2]}(2)$
$x_1^{[2]}(t)$				$L_1^{[2]}(1)$	$L_4^{[2]}(1)$	$L_7^{[2]}(1)$	$L_2^{[3]}(1)$	$L_3^{[3]}(1)$		$L_1^{[2]}(2)$	$L_4^{[2]}(2)$	$L_7^{[2]}(2)$
$x_2^{[2]}(t)$				$L_2^{[2]}(1)$	$L_5^{[2]}(1)$	$L_8^{[2]}(1)$	$L_4^{[3]}(1)$		$L_6^{[3]}(1)$	$L_2^{[2]}(2)$	$L_5^{[2]}(2)$	$L_8^{[2]}(2)$
$x_3^{[2]}(t)$				$L_3^{[2]}(1)$	$L_6^{[2]}(1)$	$L_9^{[2]}(1)$		$L_7^{[3]}(1)$	$L_8^{[3]}(1)$	$L_3^{[2]}(2)$	$L_6^{[2]}(2)$	$L_9^{[2]}(2)$
$y_1^{[3]}(t)$				$L_1^{[3]}(1)$	$L_4^{[3]}(1)$	$L_7^{[3]}(1)$	$\gamma_{12}^{[3]}(1)$	$\gamma_{13}^{[3]}(1)$		$L_1^{[3]}(2)$	$L_4^{[3]}(2)$	$L_7^{[3]}(2)$
$y_2^{[3]}(t)$				$L_2^{[3]}(1)$	$L_5^{[3]}(1)$	$L_8^{[3]}(1)$	$\gamma_{12}^{[3]}(1)$		$\gamma_{23}^{[3]}(1)$	$L_2^{[3]}(2)$	$L_5^{[3]}(2)$	$L_8^{[3]}(2)$
$y_3^{[3]}(t)$				$L_3^{[3]}(1)$	$L_6^{[3]}(1)$	$L_9^{[3]}(1)$		$\gamma_{13}^{[3]}(1)$	$\gamma_{23}^{[3]}(1)$	$L_3^{[3]}(2)$	$L_6^{[3]}(2)$	$L_9^{[3]}(2)$

the destinations. Let  $\mu_k(l)$ ,  $\nu_k(l)$ , and  $\omega_k(l)$  ( $k \in \{1, 2, 3\}$ ) denote the source messages intended for the destinations  $3_k$  (the index  $l$  means that the notations apply for the  $l$ th transmission round).

In what follows, we will explain the first round of transmission. It consists of two *phases*.

**Phase One:** The first phase takes the first 3 time slots. At time slot  $t$  ( $t \in \{1, 2, 3\}$ ),  $\mu_t(1), \nu_t(1), \omega_t(1)$  are transmitted by the three source antennas respectively. Hence each relay (i.e. each node of layer-2) receives a linear combination of three messages at each time slot. Again, we ignore the noise in (2). The received signals at  $2_k$  is expressed as ( $t \in \{1, 2, 3\}$ )

$$y_k^{[2]}(t) = h_{k1}^{[1]}(t)\mu_t(1) + h_{k2}^{[1]}(t)\nu_t(1) + h_{k3}^{[1]}(t)\omega_t(1). \quad (6)$$

Let  $L_{3(t-1)+k}^{[2]}(1) = y_k^{[2]}(t)$  denote the linear equation known by  $2_k$  at time slot  $t$ . After the 3rd time slot, since  $\mathbf{H}^{[1]}(1)$ ,  $\mathbf{H}^{[1]}(2)$ , and  $\mathbf{H}^{[1]}(3)$  are known at the source, all the equations  $L_i^{[2]}(1)$ ,  $\forall i = 1, 2, \dots, 9$ , can be recovered by the source.

**Phase Two:** This phase takes the next 6 time slots after phase one. At each time slot  $t$  ( $t \in \{3, 4, 5\}$ ) only two source antennas are activated to retransmit the equations  $L_i^{[2]}(1)$ . According to  $x_k^{[1]}(t)$  shown in Table I, we have

$$y_k^{[2]}(4) = h_{k1}^{[1]}(4)L_2^{[2]}(1) + h_{k2}^{[1]}(4)L_4^{[2]}(1); \quad (7)$$

$$y_k^{[2]}(5) = h_{k1}^{[1]}(4)L_3^{[2]}(1) + h_{k3}^{[1]}(4)L_7^{[2]}(1); \quad (8)$$



$$y_k^{[2]}(6) = h_{k2}^{[1]}(4)L_6^{[2]}(1) + h_{k3}^{[1]}(4)L_8^{[2]}(1). \quad (9)$$

Since node 2<sub>1</sub> obtains  $L_4^{[2]}(1)$  in phase one, at time slot 4 it can recover  $L_2^{[2]}(1)$ . Similarly, both  $L_2^{[2]}(1)$  and  $L_4^{[2]}(1)$  are also known at node 2<sub>2</sub>. Use  $\gamma_{ij}^{[n]}(l) = (a, b)$  to represent that equations  $a$  and  $b$  are recovered by both nodes  $n_i$  and  $n_j$ . As shown in Table I, we can replace both  $y_1^{[2]}(4)$  and  $y_2^{[2]}(4)$  with  $\gamma_{12}^{[2]}(1) = (L_2^{[2]}(1), L_4^{[2]}(1))$ . Clearly, we also have  $\gamma_{13}^{[2]}(1) = (L_3^{[2]}(1), L_7^{[2]}(1))$  and  $\gamma_{23}^{[2]}(1) = (L_6^{[2]}(1), L_8^{[2]}(1))$ .

Meanwhile, the relay nodes also send the equations they received in phase one to the destinations, as shown in Table I. The received equations at the destinations 3<sub>k</sub> are:

$$y_k^{[3]}(4) = h_{k1}^{[2]}(4)L_1^{[2]}(1) + h_{k2}^{[2]}(4)L_2^{[2]}(1) + h_{k3}^{[2]}(4)L_3^{[2]}(1), \quad (10)$$

$$y_k^{[3]}(5) = h_{k1}^{[2]}(5)L_4^{[2]}(1) + h_{k2}^{[2]}(5)L_5^{[2]}(1) + h_{k3}^{[2]}(5)L_6^{[2]}(1), \quad (11)$$

$$y_k^{[3]}(6) = h_{k1}^{[2]}(6)L_7^{[2]}(1) + h_{k2}^{[2]}(6)L_8^{[2]}(1) + h_{k3}^{[2]}(6)L_9^{[2]}(1). \quad (12)$$

Let  $L_{3(t-4)+k}^{[3]}(1) = y_k^{[3]}(t)$ . Clearly, if the destination 3<sub>1</sub> knows the three equations  $L_1^{[3]}(1)$ ,  $L_2^{[3]}(1)$ ,  $L_3^{[3]}(1)$ , it can recover its desired messages  $\mu_1(1)$ ,  $\nu_1(1)$ ,  $\omega_1(1)$ . After time slot 6, the node 3<sub>1</sub> has  $L_1^{[3]}(1)$ . Thus if  $L_2^{[3]}(1)$  and  $L_3^{[3]}(1)$  can be provided to node 3<sub>1</sub>, the problem is solved. Similarly, having  $L_5^{[3]}(1)$ , the destination 3<sub>2</sub> needs  $L_4^{[3]}(1)$  and  $L_6^{[3]}(1)$  to recover  $\mu_2(1)$ ,  $\nu_2(1)$ ,  $\omega_2(1)$ .  $L_7^{[3]}(1)$  and  $L_8^{[3]}(1)$  are desired by the destination 3<sub>3</sub>, who already has  $L_9^{[3]}(1)$ , to recover  $\mu_3(1)$ ,  $\nu_3(1)$ ,  $\omega_3(1)$ . Therefore, we aim to deliver these six equations from the relays to the destinations in the next three time slots.

According to the above description, we can see that after time slot 6, node 2<sub>1</sub> knows the equations  $L_1^{[2]}(1)$ ,  $L_2^{[2]}(1)$  and  $L_3^{[2]}(1)$ . Node 2<sub>2</sub> knows the equations  $L_4^{[2]}(1)$ ,  $L_5^{[2]}(1)$  and  $L_6^{[2]}(1)$ . Node 2<sub>3</sub> knows the equations  $L_7^{[2]}(1)$ ,  $L_8^{[2]}(1)$  and  $L_9^{[2]}(1)$ . Since the channel matrices  $\mathbf{H}^{[2]}(4)$ ,  $\mathbf{H}^{[2]}(5)$ , and  $\mathbf{H}^{[2]}(6)$  are available at all nodes in layer-2, the node 2<sub>1</sub> can formulate the equations  $L_2^{[3]}(1)$  and  $L_3^{[3]}(1)$  using (10). Similarly, the node 2<sub>2</sub> can formulate the equations  $L_4^{[3]}(1)$  and  $L_6^{[3]}(1)$  according to (11). The node 2<sub>3</sub> can formulate the equations  $L_7^{[3]}(1)$  and  $L_8^{[3]}(1)$  using (12).

At time slot 7, let 2<sub>1</sub> transmit  $L_2^{[3]}(1)$  and 2<sub>2</sub> transmit  $L_4^{[3]}(1)$ , as shown in Table I. Node 3<sub>1</sub>, which already knows  $L_1^{[3]}(1)$ , can recover  $L_2^{[3]}(1)$  by eliminating  $L_4^{[3]}(1)$  from its received signal. The node 3<sub>2</sub> can also attain both  $L_2^{[3]}(1)$  and  $L_4^{[3]}(1)$ , following the similar approach. Thus the received signals  $y_1^{[3]}(7)$  and  $y_2^{[3]}(7)$  in Table I can be replaced with a simpler expression



$\gamma_{12}^{[3]}(1) = (L_2^{[3]}(1), L_4^{[3]}(1))$ . Then we can also have  $\gamma_{13}^{[3]}(1) = (L_3^{[3]}(1), L_7^{[3]}(1))$  and  $\gamma_{23}^{[3]}(1) = (L_6^{[3]}(1), L_8^{[3]}(1))$ , at the 8th and 9th time slots, respectively.

Consequently, equations  $L_1^{[3]}(1)$ ,  $L_2^{[3]}(1)$ , and  $L_3^{[3]}(1)$  are known at the destination  $3_1$ . The desired messages can be recovered now. The same result holds also for the destinations  $3_2$  and  $3_3$ . 9 independent messages are delivered successfully from the source to the destinations in one transmission round. The same process can continue until  $l$  rounds of transmissions are finished using a total of  $6l + 3$  time slots (the second round transmission is shown in Table I partially). When  $l \rightarrow \infty$ , this scheme achieves a sum DoF  $\frac{3}{2}$ . The lower bound for  $D^{d-CSI}(3, 3)$  is proven.

To generalize this scheme to  $N$  ( $N > 3$ ) layers, we first denote the messages from the source as:  $L_{3(k-1)+1}^{[1]}(l) = \mu_k(l)$ ,  $L_{3(k-1)+2}^{[1]}(l) = \nu_k(l)$  and  $L_{3(k-1)+3}^{[1]}(l) = \omega_k(l)$ . The  $l$ th-round transmission at layer- $n$  ( $n \in \{1, 2, \dots, N-1\}$ ) can be denoted by the following formula. It takes the time slots  $t = 6(l-1) + 3(n-1) + \hat{t}$  ( $\hat{t} = 1, 2, \dots, 6$ ):

$$\mathbf{x}^{[n]}(t) = \begin{cases} \{L_{3(\hat{t}-1)+k}^{[n]}(l)\}_{k=1}^3 & \hat{t} = 1, 2, 3; \\ [L_2^{[n+1]}(l), L_4^{[n+1]}(l), 0]^T & \hat{t} = 4; \\ [L_3^{[n+1]}(l), 0, L_7^{[n+1]}(l)]^T & \hat{t} = 5; \\ [0, L_6^{[n+1]}(l), L_8^{[n+1]}(l)]^T & \hat{t} = 6. \end{cases} \quad (13)$$

Here  $\{L_{3(\hat{t}-1)+k}^{[n]}(l)\}_{i=k}^3$  represents the column vector composed by  $L_{3(\hat{t}-1)+k}^{[n]}(l)$  ( $k \in \{1, 2, 3\}$ ). We denote the received equation at node  $(n+1)_k$  when  $\hat{t} \in \{1, 2, 3\}$  as  $L_{3(\hat{t}-1)+k}^{[n+1]}(l) = \sum_{i=1}^3 h_{ki}^{[n]}(t) L_{3(\hat{t}-1)+i}^{[n]}(l)$ . By induction, we assume  $n_k$  can recover  $L_{3(k-1)+i}^{[n+1]}(l)$  after the first three time slots ( $i \in \{1, 2, 3\}$ ). Then the transmission can be designed as shown in (13) when  $\hat{t} \in \{4, 5, 6\}$ . Therefore,  $(n+1)_1$  and  $(n+1)_2$  can recover  $\gamma_{12}^{[n+1]}(l) = (L_2^{[n+1]}(l), L_4^{[n+1]}(l))$ ;  $(n+1)_1$  and  $(n+1)_3$  can recover  $\gamma_{13}^{[n+1]}(l) = (L_3^{[n+1]}(l), L_7^{[n+1]}(l))$ ; and  $(n+1)_2$  and  $(n+1)_3$  can recover  $\gamma_{23}^{[n+1]}(l) = (L_6^{[n+1]}(l), L_8^{[n+1]}(l))$  after time slot  $6l + 3(n-1)$ . Since the destinations refer to the  $N$ th layer, the  $l$ -round transmission takes  $6l + 3(N-2)$  time slots to deliver  $9l$  independent messages. The achievable sum DoF is  $\frac{9l}{6l+3(N-2)} \approx \frac{3}{2}$  when  $l \rightarrow \infty$ . The result still holds for  $K > 3$ .

Now we consider  $K = 2$ . In this case, 4 messages are delivered using 3 time slots. Let  $L_1^{[1]} = \mu_1$  and  $L_2^{[1]} = \nu_1$  denote the messages for the first destination, and  $L_3^{[1]} = \mu_2$  and  $L_4^{[1]} = \nu_2$  denote those for the second destination. During time slot 1, the nodes (or antennas)  $n_1$  and  $n_2$  ( $n \in \{1, 2, \dots, N\}$ ) send  $L_1^{[n]}$  and  $L_2^{[n]}$ , respectively. The received signal at node  $(n+1)_k$

( $k \in \{1, 2\}$ ) is  $L_k^{[n+1]} = h_{k1}^{[n]}(1)L_1^{[n]} + h_{k2}^{[n]}(1)L_2^{[n]}$ . During time slot 2,  $n_1$  sends  $L_3^{[n]}$  and  $n_2$  sends  $L_4^{[n]}$ .  $(n+1)_k$  receives  $L_{k+2}^{[n+1]} = h_{k1}^{[n]}(2)L_3^{[n]} + h_{k2}^{[n]}(2)L_4^{[n]}$ . Assume  $n_1$  can recover  $L_2^{[n+1]}$ , and  $n_2$  can recover  $L_3^{[n+1]}$ . During time slot 3,  $n_1$  and  $n_2$  transmit  $L_2^{[n+1]}$  and  $L_3^{[n+1]}$ , respectively. Since  $(n+1)_1$  knows  $L_3^{[n+1]}$ , it can recover  $L_2^{[n+1]}$ . Similarly,  $(n+1)_1$  can recover  $L_3^{[n+1]}$ . As a result, the destination  $N_1$  can thus obtain both  $\mu_1$  and  $\nu_1$  because it can have  $L_1^{[N]}$  and  $L_2^{[N]}$ . The destination  $N_2$  can obtain  $\mu_2$  and  $\nu_2$  from  $L_3^{[N]}$  and  $L_4^{[N]}$ . The achieved sum DoF is  $\frac{4}{3}$  to meet the upper bound.

## VI. CONCLUSIONS

We investigate the sum DoF of a class of multi-hop MIMO broadcast network with delayed CSIT feedback. Our results show the transmission design by treating the multi-hop network as an entity can achieve better sum DoF than the cascade approach which separates each hop individually.

## REFERENCES

- [1] M. A. Maddah-Ali and D. Tse, "Completely stale transmitter channel state information is still very useful," in *2010 Allerton Conference*.
- [2] V. R. Cadambe and S. A. Jafar, "Interference alignment and the degrees of freedom of wireless X networks," *IEEE Trans. Inform. Theory*, vol. 55, pp. 3893–3908, Sep. 2009.
- [3] M. J. Abdoli, A. Ghasemi, and A. K. Khandani, "On the degrees of freedom of K-user SISO interference and X channels with delayed CSIT." [Online]. Available: <http://arxiv.org/pdf/1109.4314>
- [4] S. W. Jeon, S. Y. Chung, and S. A. Jafar, "Degrees of freedom of multi-source relay networks," in *2009 Allerton Conference*.
- [5] T. Gou, S. A. Jafar, S. W. Jeon, and S. Y. Chung, "Aligned interference neutralization and the degrees of freedom of the 2x2x2 interference channel," *IEEE Trans. Inform. Theory*, vol. 58, pp. 4381–4395, Jul. 2012.
- [6] C. Wang, H. Farhadi, and M. Skoglund, "Achieving the degrees of freedom of wireless multi-user relay networks," *IEEE Trans. Commun.*, To appear.